GRAPH THEORY

The Four-Color Theorem Gets a Rare New Proof

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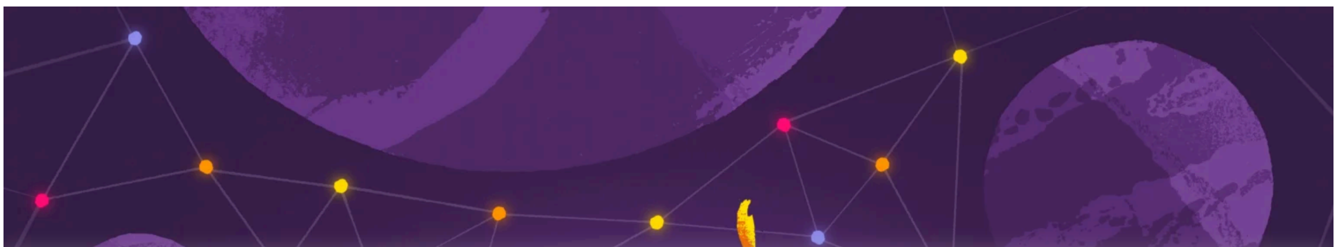
By revisiting the famous problem — which was controversially solved in the 1970s with the help of computers — mathematicians have gained important new insights into the nature of graphs.



four colors such that no neighboring regions share a color?

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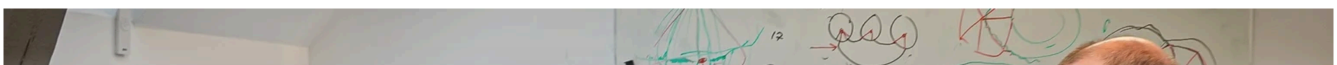
Vico Santos for *Quanta Magazine*

Some math problems continue to haunt researchers long after they've been solved. A proof emerges, is even celebrated, and yet dissatisfaction lingers. Perhaps the argument is transformed how mathematicians think about their subject.

The problem is simple to state, and even simpler to see: Given a contiguous map, is it possible to color each region with one of four colors such that no neighboring regions share a color? In the mid-19th century, the question was of trifling interest to mapmakers, who had far more than four colors at their disposal and even a child can understand," said Carsten Thomassen, a graph theorist at the Technical University of Denmark. "I think that's the reason why it has been such a big challenge."

The theorem was finally proved nearly a century later — but with computer methods that were considered scandalous at the time, causing mathematicians to question what they considered a proof in the first place. The status statement, there must be a simpler reason why it is true. Or at least a more efficient way to demonstrate it.

After nearly a decade of work, Thorup, Thomassen, and four colleagues in Denmark, Canada, and Japan have produced yet another computer proof of the theorem.



part of a team that recently re-proved the four-color theorem, revealing in the process a more efficient way to color maps and graphs.

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Courtesy of Mikkel Thorup

The proof — which was posted online in March 2026 and will be presented in November at the properties of important mathematical objects called planar graphs — opening up the potential for progress on many other stubborn problems in graph theory.

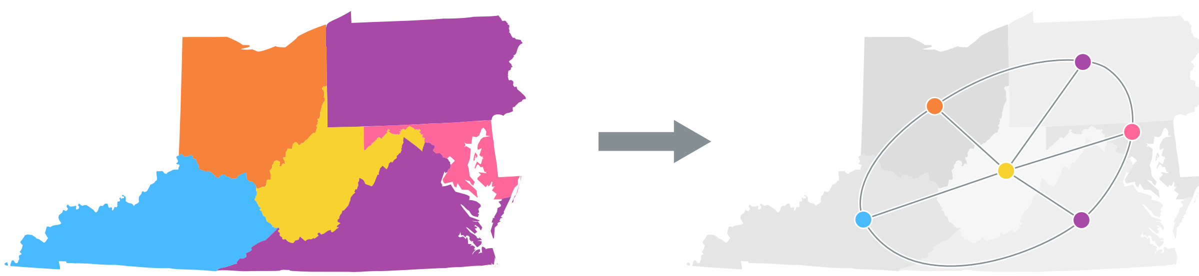
Given the problem's history of false starts and dashed hopes, Gonthier said, "it's really cool to see a real result for once."

Computerized Controversy

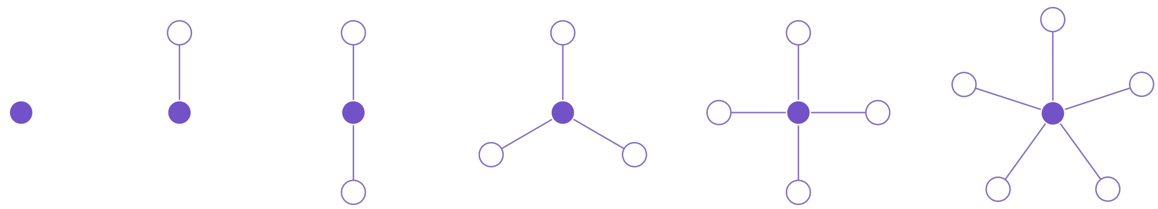
solution, a press release announced the achievement in *Nature.*

Kempe started by assuming the opposite of what he wanted to prove: that there is a map that can't be colored with only four colors. He then sought to show that this assumption would ultimately lead to a contradiction — meaning that no such map could exist. In that case, all maps would have to be four-colorable.

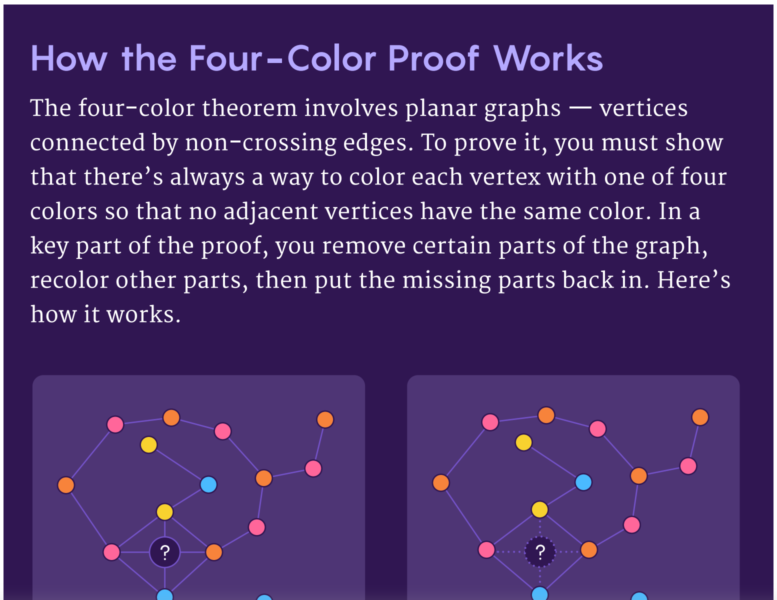
two points if those countries share a border. Your map-coloring problem is now a graph-coloring one, opening it up to the tools of graph theory.



configurations, what Kempe called an unavoidable set:



And because your graph is minimal, if you remove any of these configurations from it, you'll be left with a graph that can be recolored with four colors. Kempe's genius move was to possible to color all the vertices without needing a fifth color.

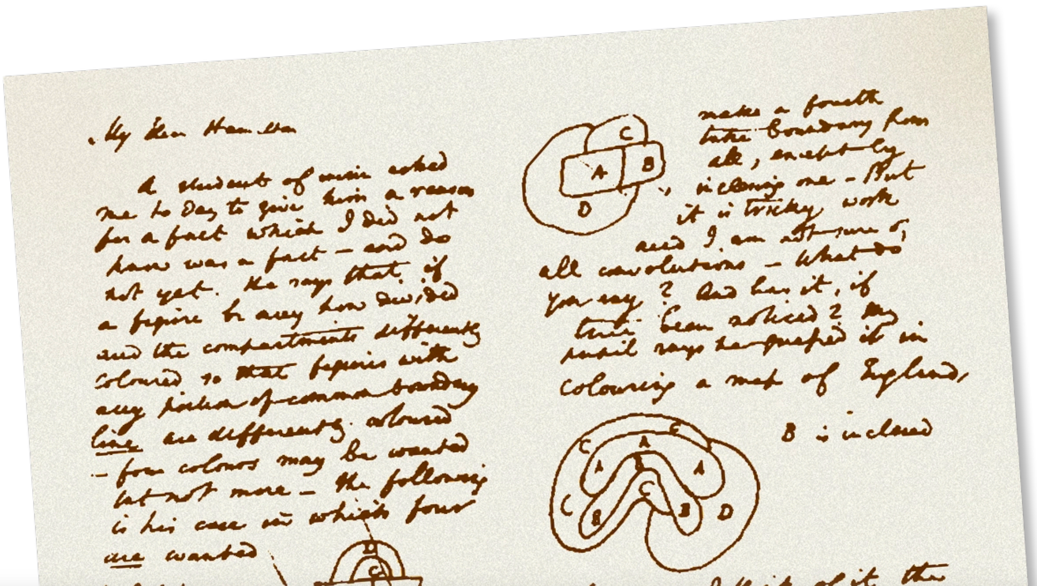


By showing that each unavoidable configuration is “reducible” in this way, you’ve demonstrated that your minimal graph is four-colorable after all — your original assumption was wrong. The four-color theorem must be true.

Unfortunately, 11 years after Kempe announced his proof, the mathematician Percy John Heawood discovered a subtle flaw in his colormistake which is so interesting that it's named after you?" Thomassen said.

In the end, no one was able to show that the last configuration in Kempe's unavoidable set was reducible. It turned out that a correct proof would instead require identifying a much larger, more complicated set of 8,900 configurations — and showing that all of them are reducible. The task was impossible to deal

Illinois to properly reduce each one. At last, they said, the four-color theorem was settled.


My dear Hamilton

A student of mine asked me to day to give him a reason for a fact which I did not know was a fact — and do not yet. He says that if a figure be any how divided and the compartments differently coloured so that figures with any portion of common boundary line are differently coloured — four colours may be wanted but not more — the following is his case in which four are wanted

make a fourth take the boundary from all, except by inclosing one — But it is tricky work and I am not sure of all convolutions — what do you say? And has it, if true been noticed? My pupil says he guessed it in colouring a map of England.

B is inclosed

me today to give him a reason for a fact which I did not know was a fact — and do not yet," he wrote in an 1852 letter to the prolific mathematician and physicist William Hamilton.

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They met a skeptical audience. Computers at the time were scary, technically unknowable. Appel and Haken were using core memory, Still, most people grew to eventually accept that "four colors suffice," as the University of Illinois later announced on their postal meter stamps. And in 1997, a team of mathematicians put the matter to bed by simplifying Appel and Haken's approach, using a computer to identify and check just 633 configurations. This time, the mathematical community accepted the result immediately.

collaborator. The pair had recently published a major paper together (which would later win them the prestigious Fulkerson Prize, also awarded decades earlier to Appel and Haken for their four-color work). They now stood on the white sand of Nyborg, wondering what to do next. “We can’t really work on a small project,” Kawarabayashi recalled thinking.

The four-color theorem had been a huge

The problem was that if you were handed some large graph and wanted to color it, you would have to search through it for one configuration, remove it, then search for another configuration, remove *that*, and so on — until you'd reduced your graph to something that was clearly four-colorable.

Kawarabayashi and Thorup, soon joined by Thomassen and Bojan Mohar of Simon Fraser

How to find these non-interfering configurations? By searching widely.

In the 1976 and 1997 proofs of the four-color theorem, mathematicians focused only on regions that featured clusters of vertices with few connections. Kawarabayashi, Mohar, Thomassen, and Thorup instead directed their attention to so-called flat areas of the graph, where every vertex is connected to six others.

Atsuyuki Miyashita (left) and Yuta Inoue helped search for important structures in regions think we had any clue it would take so long."

But eventually they landed on a new unavoidable set. It was massive, consisting of 8,202 configurations. But as they'd hoped, it was possible to reduce many of those configurations at the same time in a given graph, turning what had once been many steps into just a few.

Once again, the four-color theorem had been new insights it provides into the nature of graphs. By drawing on a different concept of reducibility and using overlooked parts of the graph, the work has revealed previously unknown structure in these important mathematical objects — and has given mathematicians new tools for understanding them. "Once you build up machinery, you find out what kind of problems you can solve," Gethner said.

questions. Though obstacles remain, Thomassen said, “I think we have the right path.”

Meanwhile, the four-color disease lives on. While Thomassen is pleased with his recent result, he still has the same question that has dogged hobbyists and professionals alike for 150 years: Is there a simpler explanation for why four colors suffice for any planar graph?